

# Supersymmetry and flavor in a sugra dual of $\mathcal{N} = 1$ Yang-Mills

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**Abstract:** Some field theory aspects can be addressed holographically by introducing D-branes in the string theory duals. In this work, we study this issue in the framework of a dual of  $\mathcal{N} = 1$  YM, the so-called Maldacena-Núñez model. Some supersymmetric embeddings for D5-brane probes are found and they are interpreted as the addition of fundamental flavors to the gauge theory. This allows to give a dual description of some known aspects of  $\mathcal{N} = 1$  SQCD and to compute a mass spectrum of mesons.

## 1 Introduction

The duality between string theories and large  $N_c$  gauge theories proposed by 't Hooft has led to surprisingly fruitful results during the last years. The best understood example relates type IIB string theory on  $AdS_5 \times S^5$  to  $\mathcal{N} = 4$  super Yang-Mills in four dimensions. Nevertheless, a large amount of work has also been devoted to the extension of the duality to less symmetric, more realistic, setups. The interest of this generalization is obvious and the final goal would be to find a string dual of large  $N_c$  QCD.

In order to develop models with reduced supersymmetry, one possibility is to consider branes wrapping supersymmetric cycles inside special holonomy manifolds. In some cases, it is possible to construct the corresponding supergravity solution. In this note, we will deal with the Maldacena-Núñez (MN) model [1], constructed along these lines.

D-branes are very useful to provide an insight of how some field theory aspects can be recognized in the dual setup. In particular, fields transforming in the fundamental representation of the gauge group must be dual to open strings with just one end on the gauge theory branes. This suggests that, in order to add flavor to a gauge theory living on a stack of  $N_c$  D-branes, one should add another stack of  $N_f$  D-branes. We will review the explicit realization of this idea in the framework of the MN model [2].

## 2 The gravity solution and its Killing spinors

The MN model was proposed as a string theory dual of  $\mathcal{N}=1$   $SU(N)$  Yang-Mills in the IR. The setup consists of a stack of  $N_c$  D5-branes extended in 1+3 Minkowski dimensions and moreover wrapped on the finite two-cycle inside a conifold. When one decouples the modes on the sphere, the open strings attached to the branes give effectively a four

dimensional gauge theory. The conifold breaks the supersymmetry to 1/4 and the branes further half it, so there are four supercharges left corresponding to one Majorana spinor in four dimensions.

The corresponding supergravity solution was computed in [3]. It realizes a geometric transition so the two-cycle shrinks and a finite three-cycle supported by RR flux is left.

The (non-singular) metric can be written in terms of the following vielbein (type IIB theory, Einstein frame):

$$\begin{aligned} e^{x^i} &= e^{\frac{\phi}{4}} dx^i \quad (i = 0, 1, 2, 3), \quad e^1 = e^{\frac{\phi}{4}+h} d\theta, \quad e^2 = e^{\frac{\phi}{4}+h} \sin\theta d\varphi, \\ e^r &= e^{\frac{\phi}{4}} dr, \quad e^{\hat{i}} = \frac{e^{\frac{\phi}{4}}}{2} (w^i - A^i) \quad (i = 1, 2, 3), \end{aligned} \quad (1)$$

where:

$$A^1 = -a(r)d\theta, \quad A^2 = a(r)\sin\theta d\varphi, \quad A^3 = -\cos\theta d\varphi. \quad (2)$$

and the  $w$ 's are  $SU(2)$  left invariant one-forms parameterizing the  $S^3$ :

$$\begin{aligned} w^1 &= \cos\psi d\tilde{\theta} + \sin\psi \sin\tilde{\theta} d\tilde{\varphi}, \quad w^2 = -\sin\psi d\tilde{\theta} + \cos\psi \sin\tilde{\theta} d\tilde{\varphi}, \\ w^3 &= d\psi + \cos\tilde{\theta} d\tilde{\varphi}. \end{aligned} \quad (3)$$

The functions appearing above and the dilaton are, explicitly:

$$a(r) = \frac{2r}{\sinh 2r}, \quad e^{2h} = r \coth 2r - \frac{r^2}{\sinh^2 2r} - \frac{1}{4}, \quad e^{-2\phi} = e^{-2\phi_0} \frac{2e^h}{\sinh 2r}. \quad (4)$$

Finally, the D5-branes are source for a RR three-form field strength:

$$F_{(3)} = -\frac{1}{4} (w^1 - A^1) \wedge (w^2 - A^2) \wedge (w^3 - A^3) + \frac{1}{4} \sum_a F^a \wedge (w^a - A^a), \quad (5)$$

where  $F^a$  is defined as an  $\mathfrak{su}(2)$  field strength:  $F^a = dA^a + \frac{1}{2}\epsilon_{abc} A^b \wedge A^c$ . In the following, we will need the explicit expressions of the Killing spinors of the solution [2]:

$$\epsilon = e^{\frac{\alpha}{2}\Gamma_1\hat{\Gamma}_1} e^{\frac{\phi}{8}} \eta, \quad (6)$$

where  $\eta$  is a constant spinor satisfying:

$$\Gamma_{x^0\dots x^3}\Gamma_{12}\eta = \eta, \quad \Gamma_{12}\eta = \hat{\Gamma}_{12}\eta, \quad \eta = i\eta^*. \quad (7)$$

Indices are referred to the frame defined in eq. (1). On the other hand,  $\alpha$  is a function of the radial variable which introduces the necessary rotation on the Killing spinor [4] and reads:

$$\cos\alpha = \coth 2r - \frac{2r}{\sinh^2 2r}. \quad (8)$$

Reviews of the model and its relation with the dual gauge theory can be found in [5].

### 3 Supersymmetric brane probes

Different gauge theory objects are described holographically by the introduction of new branes in the setup (see, for instance, [6]). In this setup, D5-branes wrapping the  $S^3$  are domain walls, fundamental strings represent QCD strings, D3-branes wrapping an  $S^2$  inside the  $S^3$  are bound states of fundamental strings and D3-branes wrapping the  $S^3$  give

the baryon vertex. By using (6), (7), one can check that the supersymmetry preserved agrees with what is expected from the field theory.

The goal of this work is to explain the introduction in the theory of dynamical quarks. This can be achieved by introducing branes extended in all Minkowski directions. They should also stretch to infinity in the holographic (radial) direction. Then, their infinite volume makes the open strings with both ends on these new branes decouple from the theory. The open strings with one end on each kind of branes are fundamental matter in the gauge theory [7]. This reasoning has also been followed to add flavor to the theories on the conifold [8] and also non supersymmetric theories [9].

In the Maldacena-Núñez model, one has to consider D5-branes extended ortogonally to the field theory branes inside the conifold [10]. In principle, one should find a new gravity solution which includes the backreaction of these new branes. However, finding this kind of backreacted solutions is in general very difficult (for recent progress in this topic in different scenarios, see [11]). Therefore, we will treat the flavor branes as probes living in the background. This, of course, limits the analysis to the quenched approximation  $N_f \ll N_c$ . The physics of the probes is described by the Born-Infeld + Wess-Zumino action, and one can study their supersymmetry using the kappa-symmetry techniques. As we know from the field theory side that quark multiplets can be introduced without further breaking supersymmetry (obtaining  $\mathcal{N} = 1$  SQCD), this will be our guiding principle.

We will require the fulfilment of:

$$\Gamma_\kappa \epsilon = \epsilon , \quad (9)$$

without introducing projections on the Killing spinor different from (7). This yields a system of first order equations for the functions defining the possible embeddings of the probes.

## 4 Holographic flavor

Equation (9) for a D5-brane probe leads, in general, to a very complicated system of equations. This problem was addressed in [2] where several solutions were found.

Let us focus on the solutions more interesting from the field theory point of view, those that can be naturally interpreted as flavor branes. These embeddings are the equivalent of the hypersurfaces considered in [12]. Let us first consider the far UV of the gauge theory. In the gravity solution, it is described by neglecting terms exponentially small in  $r$ , *i.e.* taking  $a(r) = 0$ . Then a solution of (9) is found by identifying the two-sphere and a two-sphere inside the three-sphere:

$$\tilde{\theta} = \theta , \quad \tilde{\varphi} = \varphi + \varphi_0 , \quad (10)$$

where  $\varphi_0$  is an arbitrary integration constant<sup>1</sup>. Moreover, one has to set:

$$\psi = \psi_0 , \quad e^r = \frac{e^{r_*}}{\sin \theta} \quad (a(r) = 0) , \quad (11)$$

$\psi_0$ ,  $r_*$  being two new integration constants. This embedding can be generalized when considering the full gravity solution and therefore exploring the IR of the gauge theory. Eq. (10) remains unchanged, but instead of (11), one finds:

$$\psi = \pi, 3\pi , \quad \sinh r = \frac{\sinh r_*}{\sin \theta} . \quad (12)$$

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<sup>1</sup>Another possibility is to take  $\tilde{\theta} = \pi - \theta$ ,  $\tilde{\varphi} = 2\pi - \varphi + \varphi_0$ .

These embeddings are depicted in figure 1. Let us comment on some gauge theory aspects

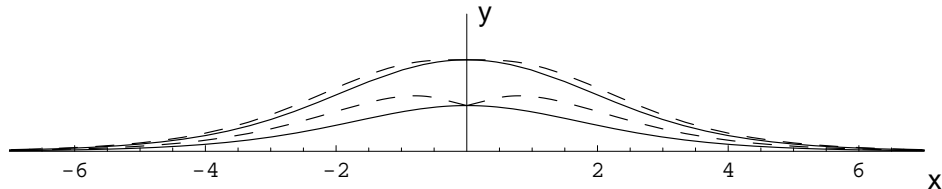


Figure 1: Pictorial representation comparing the embeddings defined in (11) (dashed line) and (12) (solid line) for the same value of  $r_*$ . The variables on each axis are  $x = r \cos \theta$ ,  $y = r \sin \theta$ . The curves for two different values of  $r_*$  ( $r_* = 0.5$  and  $r_* = 1$ ) are shown. The massless ( $r_* = 0$ ) limit would be a brane extended along the  $x$  axis at  $y = 0$ .

that can be read from the above configurations. First of all, let us analyze the global symmetries of the theory. The geometric dual of the R-symmetry of the theory is related to shifts of the  $\psi$  angle. It has been shown that the pattern of symmetry breaking of the unflavored theory is explicitly realized by the gravity solution [1]. The spontaneous breaking to  $\mathbb{Z}_2$  in the IR is related to the formation of a gaugino condensate  $\langle \lambda^2 \rangle$ , whose gravitational counterpart is the function  $a(r)$ . In the theory with flavor, there is also a squark condensate  $\langle \Phi \bar{\Phi} \rangle$  which realizes this breaking. Therefore, it is appealing to find this behavior from the probe point of view, as in the IR (12) two particular values of the constant  $\psi_0$  are selected. There is also a  $U(1)$  symmetry unbroken by the solution, related to  $\varphi_0$ . This can be identified with the baryonic  $U(1)$  of the gauge theory which stays unbroken, in agreement with a theorem by Vafa and Witten [13].

The constant of integration  $r_*$  can be identified with the mass of the quarks, as it gives the minimal distance between the gauge theory branes and the flavor branes and therefore determines the minimal energy of an open string stretching between them. This allows us to consider the massless limit by taking  $r_* \rightarrow 0$  in (12). Clearly, now the branes are extended in  $r$  at fixed  $\theta = \bar{\theta} = 0, \pi$ . But this limit is not continuous as the brane breaks into two pieces and because, moreover, one cannot consider fluctuations of the type described below. This is the holographic counterpart of the fact that, for massless flavors, there is a runaway potential in the field theory that does not allow the formation of  $\langle \lambda^2 \rangle$ ,  $\langle \Phi \bar{\Phi} \rangle$ , and therefore there is no stable vacuum. Moreover, one can check that in this limit one can again take any  $\psi = \psi_0$  as expected because there are no condensates that break the R-symmetry.

The spectrum of low energy excitations of the brane probes describes the physics of the open strings (quarks), and, therefore, one can identify it with the low energy states of the gauge theory, *i.e.* the mesons [14]. With this purpose, let us consider quadratic fluctuations around the stable embedding:

$$r(\theta, x, \varphi) = r_0(\theta) + e^{ikx} e^{il\varphi} \zeta(\theta), \quad (13)$$

where  $r_0(\theta)$  corresponds to the solution (12). By inserting this ansatz in the Born-Infeld action, one finds the lagrangian determining the dynamics of the system. Then, one would like to compute the normalizable modes which should give the quantized, physical, oscillations and get an expression for the allowed masses  $M^2 = -k^2$ . Unfortunately, this is not possible because all modes have infinite norm. This is related to the fact that the solution cannot be trusted for large  $r$  as the dilaton grows unbounded. The same problem was found when computing glueball masses in this background [15]. One can avoid this problem by introducing an UV cutoff  $r = \Lambda$  for the fluctuations. This is not unnatural

since mesons are an IR effect. The cutoff should not introduce a new scale in the setup. One should take the scale separating the IR from the UV, which coincides with the scale at which the dilaton becomes large. A precise value of  $\Lambda$  cannot be given, but it can be estimated to be around  $\Lambda \approx 3$ . One may think about this procedure as taking the difference between the oscillations around (11) and (12) which are almost identical at scales above  $\Lambda$ . Analyzing the equations of motions by means of numerical computation, we found that the mass spectrum for the scalar mesons is compatible with:

$$M_{n,l}(r_*, \Lambda) = \sqrt{m^2(r_*, \Lambda) n^2 + l^2} \quad (14)$$

where  $m(r_*, \Lambda)$  stands for the mass of the lightest meson. The quantum number  $l$  comes from the KK modes not present in the dual field theory and so it should be set to zero for the physical case. Finally, one can also find numerically the dependence of the meson masses on  $r_*$ , *i.e.* the quark mass.

$$m(r_*, \Lambda) = \frac{\pi}{2\Lambda} + b(\Lambda) r_*^2. \quad (15)$$

Our numerical results compared to the models (14), (15) are depicted in fig. 2.

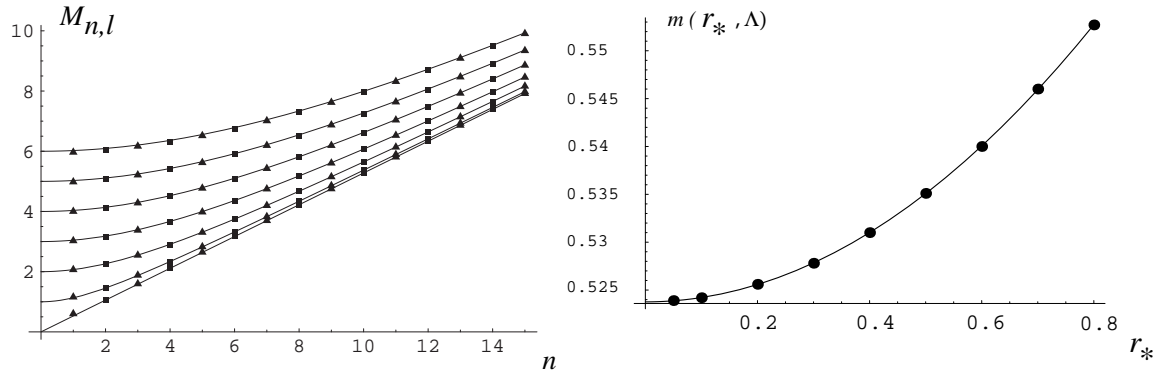


Figure 2: The spectrum of mesons found numerically. The figure on the left gives the tower of meson masses in terms of the quantum number  $n$  for different values of  $l$  ( $r_* = .3$ ,  $\Lambda = 3$ ). The one on the right depicts (for  $\Lambda = 3$ ) the lightest meson mass in terms of  $r_*$ , which can be identified with the mass of the dynamical quarks. The solid lines are given by the equations (14), (15) respectively.

It is also worth to point out that it is also possible to deal with vector mesons by considering excitations of the gauge field living on the brane probes. The relevant ansatz is:

$$\mathcal{A}_\mu(\theta, x, \varphi) = \epsilon_\mu \zeta(\theta) e^{ikx} e^{il\varphi}, \quad (16)$$

where  $\epsilon_\mu$  is a constant polarization four-vector. The numerical computation yields an spectrum almost identical to the one computed for scalar mesons (14), (15) and therefore the model predicts a degeneracy between both kinds of states. This fact should be related to the supersymmetry of the theory living on the branes.

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